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MODELS FOR BALLISTIC WIND MEASUREMENT ERROR ANALYSIS
VOLUME I: MODEL FORMULATION

JUNE 1982

By

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Under Contract DAAD07-79-C-0008

Contract Monitor: Bernard F. Engebos

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US Army Electronics Research and Development Command
Atmospheric Sciences Laboratory
White Sands Missile Range, NM 88002

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Three models for ballistic wind measurement error analysis are discussed. These models, which were originally formulated by Donald M. Swingle, are named RAWIN, RADAR, and NAVAID. Each is applicable to a different type of meteorological acquisition system. RAWIN models the case of a balloon-borne radiosonde and ground-based set for radiodirection finding and telemetry data reception. RADAR models the case in which a ground-based radar set tracks an

20. ABSTRACT (cont)

ascending balloon. NAVAID models the case in which radionavigation techniques are used to determine radiosonde position.

Expressions for the variance in the East and North components of ballistic wind are obtained in terms of bias and random measurement errors and other parameters. Also, an average error quantity called the component velocity variance is defined.

In Volume I each model is described, and the necessary computational expressions are derived. In Volume II the utilization of the associated computer programs on the UNIVAC 1108 at White Sands Missile Range is described.

ACKNOWLEDGEMENTS

The measurement error analysis models discussed in this report were originally formulated by Donald M. Swingle for use in cost operational effectiveness analyses of competitive meteorological data acquisition systems. Numerous discussions with him were helpful in the development of this presentation.

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The author also wishes to acknowledge Douglas Anderson, who did the majority of the computer programming. William Shuster also helped in this regard.

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1. INTRODUCTION

In order to achieve accurate artillery fire, the net effect of winds on the projectile at various altitudes must be taken into consideration. One aspect of quantifying this effect involves the computation of ballistic wind velocity from data collected by any of several types of meteorological (MET) data acquisition systems. Inherent errors in measurement propagate to produce an undesirable error in the computed value of ballistic wind, since the measurement errors depend on the particular type of MET system employed. It is useful to have methods by which the performance of various systems of interest can be compared.

In this report, three error analysis models are presented, each of which is applicable to one general type of MET data system. Each system modeled measures a particular set of variables to determine successive positions of a balloon-borne radiosonde as it ascends through the atmosphere. Known or estimated errors in the geometric variables, an actual or postulated wind profile, and certain other parameters are inputted to the appropriate model. From this information the model computes, among other results, a quantity called the component velocity variance (CVV) of the ballistic wind.

For a given direction of artillery fire, the ballistic wind can be resolved into range wind and crosswind components. The CVV is obtained by averaging the variance associated with either of these components over all possible directions of fire in the horizontal plane. The CVV represents a general error quantity that can be used as a basis for comparing MET systems or for evaluating suggested improvements in them.

The following general assumptions are made: (1) the CVV exists and is a useful characterization of the measuring system, and (2) first-order error analysis is sufficient to determine the CVV to an acceptable accuracy. Although not absolutely necessary, it is convenient to assume that errors in measurement are normally distributed. Further assumptions are noted later as they are required.

The error analysis models are outlined below:

(1) RAWIN models the case of a balloon-borne radiosonde and a ground-based set for radiodirection finding (RDF) and telemetry data reception. This model is applicable, for example, to the Rawin Set AN/GMD-1. It is also applicable to the Meteorological Data System AN/TMQ-31, operating in the RDF mode; this set is also known as the Field Artillery Meteorological Acquisition System (FAMAS).

(2) RADAR models the case in which a ground-based radar measures all the variables required to determine the balloon's position. This model is applicable, for example, to the radar sets AN/TMQ-19 and AN/FPS-16.

(3) NAVAID models the case in which radionavigation techniques are used to determine the position of the radiosonde. This model is applicable to the radionavigation portions (LORAN, VLF, and OMEGA) of the AN/TMQ-31.

The models represent a compromise between extreme generality and overt specialization to any particular field environment.

Material relevant to all three models is presented in Sections 2, 3, 4, and 8 of this report. Sections 5, 6, and 7 discuss specialization to each of the models, respectively. Part of the mathematical treatment is relegated to the appendices. Computer programs and sample calculations are given in the accompanying User's Manual.

2. PRELIMINARY CONSIDERATIONS

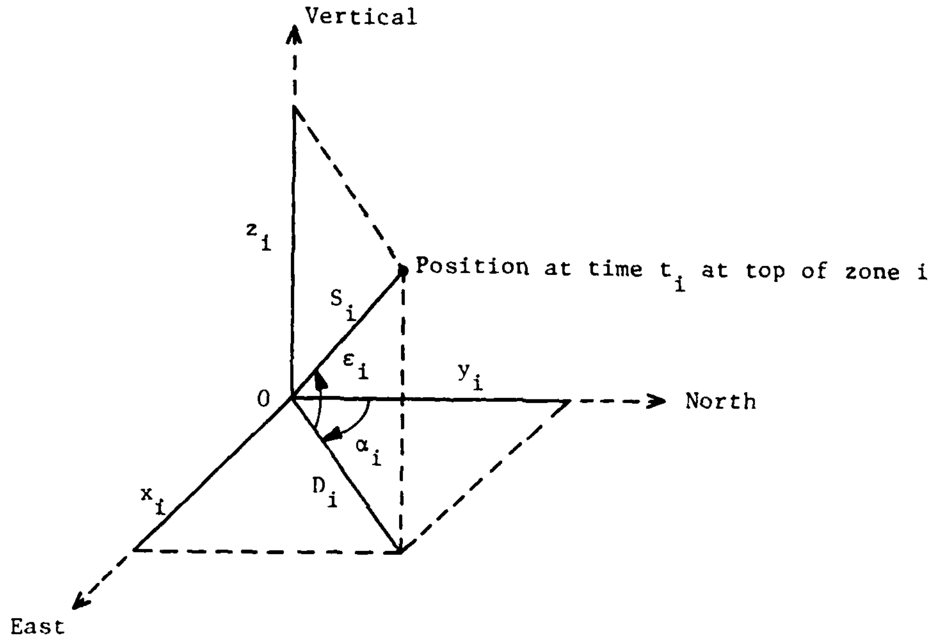
For the purpose of computing ballistic wind, the atmosphere is divided into a series of zones, not all of which are necessarily of the same vertical thickness. Ballistic line n is associated with the path followed by a round that attains its maximum altitude at the top of the n'th zone. The components of ballistic wind for line n can be determined from the average wind components in each of the first n zones in conjunction with a set of zone wind weighting factors appropriate to line n. Background information on the various accepted zone structures and weighting factors is available in References 1 and 2.

In either firing table or computer gunnery, the weighting factors given in Reference 2 are believed to be a good representation of meteorological effects on commonly used surface-to-surface and surface-to-air ballistic projectiles. However, should the user desire, he may readily substitute other appropriate weighting factors, which can be computed from results of ballistic simulation programs.

To obtain the data required to compute the ballistic wind, a balloon-borne radiosonde is tracked. In the error models presented here, it is assumed that the radiosonde, actual or postulated, ascends at a constant rate. It is also assumed that the average velocity of the wind in any zone is the same as the average horizontal velocity of the radiosonde package as it traverses the zone. This is reasonable because of the relatively large area of a MET balloon and the relatively small mass of a balloon-radiosonde combination. As is customary among physicists, engineers, and mathematicians, we take the wind direction to be that toward which the wind is blowing; this sense is opposite to the usual meteorological convention. (The computed values of the CVV are not affected by the choice of wind convention.)

Figure 1 illustrates the geometric variables associated with the radiosonde's position at the top of the i'th ballistic zone. In the RAWIN and RADAR models, the ground-based RAWIN or RADAR set defines the origin of coordinates; in NAVAID an arbitrary origin can be chosen. At time t_i the radiosonde is at altitude z_i at the top of zone i; its elevation angle is ε_i, and its azimuth angle is α_i; x_i and y_i are, respectively, the East and North components of the

Figure 1. Geometric Variables



radiosonde's position; D_i is the distance along the surface of the earth from the origin to a point directly below the radiosonde; S_i is the slant range. For ballistic line n, the zone index i takes on the values 1, 2, ..., n; the value i = 0 corresponds to the launch position and is treated as a special case. The set of variables taken as independent (or even utilized) depends on the particular model. For simplicity of presentation, Figure 1 illustrates the variables for the case of a flat earth; however, in all three error analysis models, computations are carried out assuming a spherical earth.

All three models calculate the variance associated with each component (East and North) of ballistic wind prior to finding the CVV. These variances are computed from the following inputs:

(1) The zone structure and appropriate weighting factors
(2) The ascent rate of the balloon
(3) The zone wind profile
(4) The successive values of the independent geometric variable determining the position of the radiosonde
(5) The errors associated with these variables

The input errors are taken to be standard deviations. Explanations of all required inputs are tabulated in the sections of this report that deal with the individual models.

In RAWIN and NAVAID, errors associated with the determination of altitude must be estimated beforehand or calculated from actual pressure and temperature data; no provision exists in the models for dealing directly with pressure and temperature measurements. See Reference 3, for *example*.

In RAWIN and RADAR, errors associated with the measurement of elevation angle are divided into two groups: (1) errors in elevation due to reflection of the incoming signal at the surface of the earth, and (2) errors in elevation due to all other causes. The latter group is referred to as the errors in the elevation tracking of the apparent target; for brevity it is also referred to here simply as "leveling" errors. The leveling errors can be estimated or obtained from equipment manufacturer's specifications, while the reflection errors must be estimated or computed externally to the models for given soil types and signal frequencies.

It is assumed that any uncertainty in the actual measurement of time is negligible. However, because t_i represents the elapsed time as the balloon passes the top of zone i, it contains some error because there is error in the determination of altitude. Since the balloon has a constant ascent rate, the error in t_i is directly related to the error in altitude. In actuality, the models use time only implicitly as a parameter that establishes a correspondence

among the geometric variables. In RAWIN, for example, this simply means that values of altitude, elevation, and azimuth are available as the radiosonde passes each zone top.

Two general categories of error are treated: (1) bias errors, which are correlated from zone to zone; and (2) random errors, which are uncorrelated from zone to zone. These are discussed further in Section 4.

The models are programmed to accept the NATO zone structure of 15 ballistic zones as input, along with the necessary zone wind weighting factors. However, the models can be modified to accept an arbitrary zone structure containing up to 30 zones, along with appropriate weighting factors. This modification is explained in the accompanying Users' Manual. Also, each model will accept up to four balloon ascent rates simultaneously.

In performing intermediate computations, each model converts all inputs to meter-kilogram-second (MKS) units. Many of the intermediate results are optionally available as output. The CVV itself is computed for each ballistic line for each balloon ascent rate and is outputted in knots squared. The square root of the CVV is called the standard deviation and is outputted in knots.

In the following sections of this report, various equations required by the models are derived for the case of a single ballistic line and a single balloon ascent rate. Unless otherwise noted, all variables and constants used in these derivations are considered to be in MKS units. Extension to additional ballistic lines and ascent rates is straightforward.

3. THE BALLISTIC WIND

The three error models described in this report make common use of certain algebraic expressions involving the components of ballistic wind. In this section, these expressions are derived for the East component. Equations applicable to the North component can be obtained in an analogous fashion. In all the equations, it is implicitly recognized that the zone index i takes on all values, $i = 1, 2, \ldots, n$, appropriate to ballistic line n. The launch index, i.e., $i = 0$, is treated separately as a special case.

Let u_i and v_i be, respectively, the East and North components of the average wind velocity in zone i. They are defined by the expressions

$$u_i = \frac{x_i - x_{i-1}}{t_i - t_{i-1}} \quad , \tag{3-1}$$

and

$$v_i = \frac{y_i - y_{i-1}}{t_i - t_{i-1}} \quad , \tag{3-2}$$

where x_i and y_i are, respectively, the East and North coordinates of the radiosonde at time t_i at the top of zone i.

The ballistic wind is that single value that is equivalent to the cumulative effect of the individual zone winds. The East component U of the ballistic wind is defined by:

$$U = \sum_{i=1}^{n} w_i u_i \quad , \tag{3-3}$$

where w_i is the wind weighting factor appropriate to the i'th zone for ballistic line n. Then,

$$U = \frac{w_1(x_1 - x_0)}{t_1 - t_0} + \frac{w_2(x_2 - x_1)}{t_2 - t_1} + \ldots + \frac{w_i(x_i - x_{i-1})}{t_i - t_{i-1}}$$

$$+ \frac{w_{i+1}(x_{i+1} - x_i)}{t_{i+1} - t_i} + \ldots + \frac{w_n(x_n - x_{n-1})}{t_n - t_{n-1}} \quad , \tag{3-4}$$

where x_0 is the East coordinate of launch position, and t_0 is the launch time. Collecting terms in x_i yields

$$U = \left[- \frac{w_1}{t_1 - t_0} \right] x_o + \left[\frac{w_1}{t_1 - t_0} - \frac{w_2}{t_2 - t_1} \right] x_1 + \ldots$$

$$+ \left[\frac{w_i}{t_i - t_{i-1}} - \frac{w_{i+1}}{t_{i+1} - t_i} \right] x_i + \ldots + \left[\frac{w_n}{t_n - t_{n-1}} \right] x_n \quad , \tag{3-5}$$

or

$$U = \left[- \frac{w_1}{t_1 - t_0} \right] x_0 + \sum_{i=1}^{n} \left[\frac{w_i}{t_i - t_{i-1}} - \frac{w_{i+1}}{t_{i+1} - t_i} \right] x_i \quad , \tag{3-6}$$

where $w_{n+1} = 0$ for ballistic line n.

For a constant ascent rate v_z, the time spent by the balloon in zone i is given by:

$$t_i - t_{i-1} = \frac{z_i - z_{i-1}}{v_z} \quad , \tag{3-7}$$

where z_i is the altitude at the top of the i'th zone. Combining Eq. (3-7) with Eq. (3-6), we have

$$U = v_z \left[\frac{w_1}{z_1 - z_0} \right] x_0 + \sum_{i=1}^{n} v_z \left[\frac{w_i}{z_i - z_{i-1}} - \frac{w_{i+1}}{z_{i+1} - z_i} \right] x_i \ . \tag{3-8}$$

The set of altitudes z_i defines the zone structure and is available in the form of tabulated values. From these values, it is convenient to define for ballistic line n a set of new constant weighting factors (per unit length), W_0 and W_i, that depend only on the zone structure:

$$W_0 = - \frac{w_1}{z_1 - z_0} \quad ; \tag{3-9}$$

$$W_i = \frac{w_i}{z_i - z_{i-1}} - \frac{w_{i+1}}{z_{i+1} - z_i} \quad . \tag{3-10}$$

Then Eq. (3-8) becomes

$$U = v_z W_0 x_0 + \sum_{i=1}^{n} v_z W_i x_i \tag{3-11}$$

The partial derivatives $\frac{\partial U}{\partial x_i}$ will be required to compute the variance in the East component of ballistic wind. From Eq. (3-11) these are:

$$\frac{\partial U}{\partial x_0} = v_z W_0 \tag{3-12}$$

and

$$\frac{\partial U}{\partial x_i} = v_z W_i \quad . \tag{3-13}$$

The derivatives $\frac{\partial U}{\partial t_i}$ are also required. To compute these, we consider only those terms of Eq. (3-4) that contain the general zone index i explicitly:

$$U = \ldots + \frac{w_i(x_i - x_{i-1})}{t_i - t_{i-1}} + \frac{w_{i+1}(x_{i+1} - x_i)}{t_{i+1} - t_i} + \ldots \tag{3-14}$$

Then,

$$\frac{\partial U}{\partial t_i} = -\frac{w_i(x_i - x_{i-1})}{(t_i - t_{i-1})^2} + \frac{w_{i+1}(x_{i+1} - x_i)}{(t_{i+1} - t_i)^2} \quad . \tag{3-15}$$

Making use of the relationships expressed in Eqs. (3-1) and (3-7), we rewrite Eq. (3-15) as:

$$\frac{\partial U}{\partial t_i} = \left[\frac{w_{i+1}}{z_{i+1} - z_i} u_{i+1} - \frac{w_i}{z_i - z_{i-1}} u_i \right] v_z \quad . \tag{3-16}$$

We could also find $\frac{\partial U}{\partial t_0}$, but an explicit determination of this quantity is not required by the error analysis models.

From Eq. (3-7) we have

$$\frac{\partial t_i}{\partial z_i} = v_z \quad . \tag{3-17}$$

The models make use of the product of derivatives $\frac{\partial U}{\partial t_i} \frac{\partial t_i}{\partial z_i}$. From Eqs. (3-16) and (3-17) we have

$$\frac{\partial U}{\partial t_i} \frac{\partial t}{\partial z_i} = \frac{w_{i+1}}{z_{i+1} - z_i} u_{i+1} - \frac{w_i}{z_i - z_{i-1}} u_i \quad . \tag{3-18}$$

For convenience we write this more briefly as

$$\frac{\partial U}{\partial t_i} \frac{\partial t}{\partial z_i} = W_{ui} \quad , \tag{3-19}$$

where

$$W_{ui} = \frac{w_{i+1}}{z_{i+1} - z_i} u_{i+1} - \frac{w_i}{z_i - z_{i-1}} u_i \tag{3-20}$$

Similar expressions involving V, the North component of ballistic wind, can also be derived. The models make use of the following:

$$\frac{\partial V}{\partial y_0} = v_z W_0 \quad , \tag{3-21}$$

$$\frac{\partial V}{\partial y_i} = v_z W_i \quad , \tag{3-22}$$

and

$$\frac{\partial V}{\partial t_i} \frac{\partial t_i}{\partial z_i} = W_{vi} \quad , \tag{3-23}$$

where

$$W_{vi} = \frac{w_{i+1}}{z_{i+1} - z_i} v_{i+1} - \frac{w_i}{z_i - z_{i-1}} v_i \quad . \tag{3-24}$$

4. FORM OF THE CVV

The variance in either component of ballistic wind can be written in a general form that is easily specialized to each of the error models under consideration. In this section, we examine this form and the associated assumptions concerning bias and random error. Finally, we obtain the expression for the CVV.

From Eq. (3-4), the East component U of ballistic wind for ballistic line n can be written in the implicit functional form

$$U = U\,[x_0,\ t_0,\ x_i,\ t_i],\quad i = 1, 2, \ldots n \quad . \tag{4-1}$$

In general, each of the variables x_0, t_0, x_i, and t_i may themselves be functions of other variables. In RAWIN, for example, the East component x_0 of launch position is taken to be a function of D_0 and α_0, where D_0 is the distance from the Rawin Set to the launch site, and α_0 is the launch site azimuth. Also in RAWIN, the ascending radiosonde's East component x_i is considered to be a function of the altitude z_i, elevation ε_i, and azimuth α_i, all determined for the top of zone i. In all the models, t_i is taken to be a function of z_i through Eq. (3-7).

In general, we can rewrite Eq. (4-1) as

$$U = U\,[\lambda_{10},\ \lambda_{20},\ \ldots,\ \lambda_{L0},\ \xi_{1i},\ \xi_{2i},\ \ldots,\ \xi_{ki}],\ i = 1, 2, \ldots, n\ , \tag{4-2}$$

where λ_{10}, λ_{20}, ..., λ_{L0} is the model-dependent subset of independent launch variables, and ξ_{1i}, ξ_{2i}, ..., ξ_{ki} comprise the model-dependent subset of independent variables appropriate to zone i.

For a given independent variable, two categories of measurement error are considered: random error and bias error. The random errors of measurement follow some distribution law (which for convenience may be taken to be normal) with zero mean and a characteristic variance and standard deviation.

The bias error in a given variable may arise from inaccurate calibration of the measuring device, causing the determined values to be always too low or too high for a particular experiment. However, over many experiments, the calibration of an instrument is as likely to be overperformed as under-performed. From this standpoint, the bias errors themselves are random and have some distribution (which for convenience may be assumed to be normal) with zero mean and a characteristic variance and standard deviation. This is the point of view adopted in this report. Bias and random errors are treated differently here only in the assumptions made concerning their zone-to-zone correlations.

We expect no correlation among the variables ξ_{ki} and ξ_{ji}, $i = 1, 2, \ldots, n$, $j \neq k$, because values of these variables are determined from completely different types of measurement. For example, ξ_{1i}, ξ_{2i}, and ξ_{3i} might represent z_i, ε_i, and α_i, respectively.

It is assumed that errors in the measurement of any one type of zone variable ξ_{ki}, $i = 1, 2, \ldots, n$ can be treated in the following manner: random errors are uncorrelated from zone to zone; bias errors are completely correlated from zone to zone. In other words, the random errors have correlation coefficients of zero, while the bias errors have correlation coefficients of one.

Finally, we assume only random errors in the determination of the independent launch variables λ_{10}, λ_{20}, $\ldots$, λ_{L0}.

If only first-order propagation of error is considered, it is shown in Appendix A that the variance σ_U^2 in the East component of ballistic wind for line n is given by:

$$\sigma_U^2 = \sum_{k=1}^{L} \left[\frac{\partial U}{\partial \lambda_{k0}} \sigma_{R\lambda k0} \right]^2 + \sum_{k=1}^{K} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \xi_{ki}} \sigma_{B\xi ki} \right]^2 + \sum_{k=1}^{K} \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \xi_{ki}} \sigma_{R\xi ki} \right]^2 . \tag{4-3}$$

In Eq. (4-3), $\sigma_{R\lambda k0}$ is the standard deviation associated with random errors in the measurement of λ_{k0}, while $\sigma_{B\xi ki}$ and $\sigma_{R\xi ki}$ are the standard deviations associated respectively with the bias and random errors in the measurement of ξ_{ki}. In computation, the bias contributions are summed over the appropriate zones before being squared, while the random contributions are squared before being summed. Similarly, the variance σ_V^2 in the North component V of ballistic wind is

$$\sigma_V{}^2 = \sum_{k=1}^{L} \left[\frac{\partial V}{\partial \lambda_{k0}} \sigma_{R\lambda k0} \right]^2 + \sum_{k=1}^{K} \left[\sum_{i=1}^{n} \frac{\partial V}{\partial \xi_{ki}} \sigma_{B\xi ki} \right]^2 + \sum_{k=1}^{K} \sum_{i=1}^{n} \left[\frac{\partial V}{\partial \xi_{ki}} \sigma_{R\xi ki} \right]^2 . \tag{4-4}$$

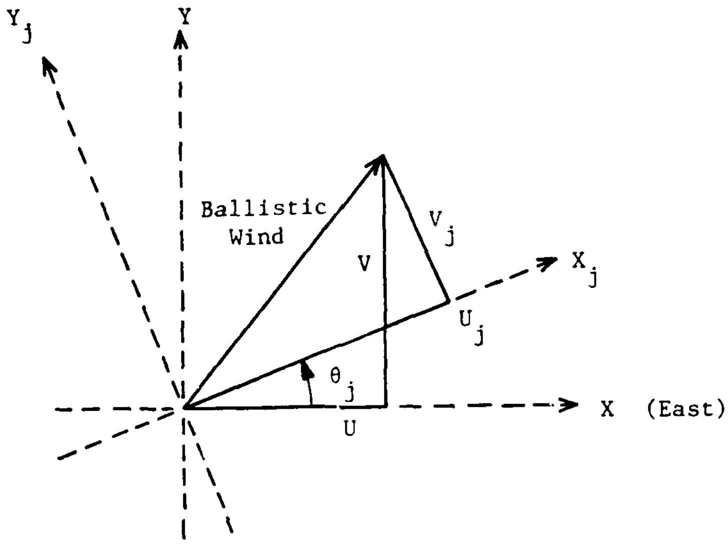
For a given direction of artillery fire, the ballistic wind can be resolved into range wind and crosswind components. The CVV is obtained by averaging the variance associated with either of these components over all possible directions of fire in the horizontal plane.

Let the j'th direction of artillery fire make an angle θ_j with the positive X axis or East direction. We define in the horizontal plane an orthogonal coordinate system, labeled X_jY_j, such that the positive X_j axis points in the (horizontal) direction of fire. This coordinate system is depicted in Figure 2. Each possible direction of fire corresponds to a particular orientation of the X_jY_j system.

The X_j component of ballistic wind is the range wind U_j, and the Y_j component is the crosswind V_j, as shown in Figure 2. From a standard transformation of coordinates, it can be shown that

$$U_j = U \cos \theta_j + V \sin \theta_j \quad , \tag{4-5}$$

Figure 2. Orientation of Axes



The variance $\sigma_{U_j}^2$ appropriate to U_j is given by

$$\sigma_{U_j}^2 = \left(\frac{\partial U_j}{\partial U}\right)^2 \sigma_U^2 + \left(\frac{\partial U_j}{\partial V}\right)^2 \sigma_V^2 + 2 \frac{\partial U_j}{\partial U} \frac{\partial U_j}{\partial V} \sigma_{UV}^2 \quad , \qquad (4\text{-}6)$$

where σ_{UV}^2 is the covariance between U and V. From Eqs. (4-5) and (4-6) we have

$$\sigma_{U_j}^2 = \sigma_U^2 \cos^2 \theta_j + \sigma_V^2 \sin^2 \theta_j + 2 \sigma_{UV}^2 \cos \theta_j \sin \theta_j \quad . \qquad (4\text{-}7)$$

For a large number N of equally spaced orientations of axes, the CVV σ_c^2 is defined by

$$\sigma_c^2 = \frac{1}{N} \sum_{j=1}^{N} \sigma_{Uj}^2 \tag{4-8}$$

or

$$\sigma_c^2 = \frac{1}{N} \sum_{j=1}^{N} [\sigma_U^2 \cos^2 \theta_j + \sigma_V^2 \sin^2 \theta_j + 2\, \sigma_{UV}^2 \cos \theta_j \sin \theta_j] \;. \tag{4-9}$$

If $\Delta\theta$ is the constant angular displacement between successive orientations of axes, then the total number of orientations over 2π is given by

$$N = \frac{2\pi}{\Delta\theta} \;. \tag{4-10}$$

Thus, Eq. (4-9) can be written as

$$\sigma_c^2 = \frac{\sigma_U^2}{2\pi} \sum_{j=1}^{N} \cos^2 \theta_j \,\Delta\theta + \frac{\sigma_V^2}{2\pi} \sum_{j=1}^{N} \sin^2 \theta_j \,\Delta\theta + \frac{\sigma_{UV}^2}{\pi} \sum_{j=1}^{N} \cos \theta_j \sin \theta_j \,\Delta\theta \tag{4-11}$$

In the limit of small $\Delta\theta$, i.e., large N, we can replace the sums in Eq. (4-11) by integrals:

$$\sigma_c^2 = \frac{\sigma_U^2}{2\pi} \int_0^{2\pi} \cos^2 \theta \, d\theta + \frac{\sigma_V^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta$$

$$+ \frac{\sigma_{UV}^2}{\pi} \int_0^{2\pi} \cos \theta \sin \theta \, d\theta \quad . \tag{4-12}$$

Each of the first two integrals yields π, while the third integral yields zero. Then

$$\sigma_c^2 = \frac{1}{2} \sigma_U^2 + \frac{1}{2} \sigma_V^2 \quad , \tag{4-13}$$

The quantity σ_c^2 is the CVV that is referred to elsewhere in this report. It is used to characterize the error behavior of each MET data system.

5. RAWIN

RAWIN models the case of a balloon-borne radiosonde and a ground-based set for RDF and telemetry data reception. The angular variables of the radiosonde's position are measured by the RDF ground set, and the altitude can be determined from MET messages received from the radiosonde. This model is applicable, for example, to the RAWIN set AN/GMD-1 and to the MET data system AN/TMQ-31 (FAMAS) operating in the RDF mode.

In RAWIN, the radiosonde's East coordinate x_i is assumed to be a function of the independent variables of altitude z_i, elevation angle ε_i, and azimuth angle α_i, all determined for the top of the i'th ballistic zone. These variables are illustrated in Figure 1. The time t_i at which the radiosonde passes the top of the zone is taken to be a function of z_i. The balloon is released at time t_0. The East coordinate x_0 of launch position is taken to be a function of D_0 and α_0, where D_0 is the distance from the receiving set to the launch site, and α_0 is the associated azimuth.

Under the conditions noted above, Eq. (4-1) may be written in the slightly more explicit functional form:

$$U = U\,[x_0\ (D_0,\ \alpha_0),\ t_0,\ x_i\ (z_i,\ \varepsilon_i,\ \alpha_i),\ t_i\ (z_i)],\ i = 1,\ 2,\ \ldots\ n, \tag{5-1}$$

where U is the East component of ballistic wind appropriate to line n. A similar equation holds for the North component.

To obtain the variance $\sigma_U{}^2$ associated with U, we rewrite Eq. (4-3) in terms of the independent variables D_0, α_0, z_i, ε_i, and α_i. This yields

$$\sigma_U{}^2 = [(\frac{\partial U}{\partial D_0}\ \sigma_{RD0})^2 + (\frac{\partial U}{\partial \alpha_0}\ \sigma_{R\alpha 0})^2] +$$

$$+ [\sum_{i=1}^{n} \frac{\partial U}{\partial z_i} \sigma_{Bzi}]^2 + [\sum_{i=1}^{n} \frac{\partial U}{\partial \varepsilon_i} \sigma_{B\varepsilon iL}]^2 + [\sum_{i=1}^{n} \frac{\partial U}{\partial \varepsilon_i} \sigma_{B\varepsilon iG}]^2 + [\sum_{i=1}^{n} \frac{\partial U}{\partial \alpha_i} \sigma_{B\alpha i}]^2$$

(5-2)

$$+ \sum_{i=1}^{n} [\frac{\partial U}{\partial z_i} \sigma_{Rzi}]^2 + \sum_{i=1}^{n} [\frac{\partial U}{\partial \varepsilon_i} \sigma_{R\varepsilon iL}]^2 + \sum_{i=1}^{n} [\frac{\partial U}{\partial \varepsilon_i} \sigma_{R\varepsilon iG}]^2 + \sum_{i=1}^{n} [\frac{\partial U}{\partial \alpha_i} \sigma_{R\alpha i}]^2 \; .$$

In Eq. (5-2), each type of error is designated by σ with appropriate subscripts. The first subscript, B or R, identifies the error as bias or random, respectively. The second subscript is the variable containing the error. The third subscript is 0 for the launch variables; otherwise, it is the zone index i. For errors in the measurement of elevation, a fourth subscript is present: G identifies the error associated with the reflection of the transmitted signal by the ground; L identifies the combined error in elevation due to all other causes. Thus, for example, $\sigma_{R\varepsilon iG}$ is the random error in the measurement of elevation at zone top i due to ground reflection. It is assumed that the launch time is known precisely; hence, an error term involving t_0 is not included in Eq. (5-2).

It is assumed that elevation errors due to ground reflection are not correlated with elevation errors due to other causes. Thus they enter independently into the computation of σ_U^2 in the manner shown in Eq. (5-2).

Analogous to Eq. (5-2), the expression for the variance σ_V^2 in the North component V of ballistic wind is

$$\sigma_V^2 = [(\frac{\partial V}{\partial D_0} \sigma_{RD0})^2 + (\frac{\partial V}{\partial \alpha_0} \sigma_{R\alpha 0})^2]$$

$$+ [\sum_{i=1}^{n} \frac{\partial V}{\partial z_i} \sigma_{Bzi}]^2 + [\sum_{i=1}^{n} \frac{\partial V}{\partial \varepsilon_i} \sigma_{B\varepsilon iL}]^2 + [\sum_{i=1}^{n} \frac{\partial V}{\partial \varepsilon_i} \sigma_{B\varepsilon iG}]^2 + [\sum_{i=1}^{n} \frac{\partial V}{\partial \alpha_i} \sigma_{B\alpha i}]^2 +$$

$$+ \sum_{i=1}^{n} [\frac{\partial V}{\partial z_i} \sigma_{Rzi}]^2 + \sum_{i=1}^{n} [\frac{\partial V}{\partial \varepsilon_i} \sigma_{R\varepsilon iL}]^2 + \sum_{i=1}^{n} [\frac{\partial V}{\partial \varepsilon_i} \sigma_{R\varepsilon iG}]^2 + \sum_{i=1}^{n} [\frac{\partial V}{\partial \alpha_i} \sigma_{R\alpha i}]^2 \quad . \tag{5-3}$$

In order to obtain computationally useful error coefficients, each of the partial derivatives in Eqs. (5-2) and (5-3) must be further expanded. The form of Eq. (5-1) allows for straightforward use of the chain rule for partial derivatives to achieve this. We find that

$$\frac{\partial U}{\partial D_0} = \frac{\partial U}{\partial x_0} \frac{\partial x_0}{\partial D_0} \quad , \tag{5-4}$$

$$\frac{\partial U}{\partial \alpha_0} = \frac{\partial U}{\partial x_0} \frac{\partial x_0}{\partial \alpha_0} \quad , \tag{5-5}$$

$$\frac{\partial U}{\partial z_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial z_i} + \frac{\partial U}{\partial t_i} \frac{\partial t_i}{\partial z_i} \quad , \tag{5-6}$$

$$\frac{\partial U}{\partial \varepsilon_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial \varepsilon_i} \quad , \tag{5-7}$$

and

$$\frac{\partial U}{\partial \alpha_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} \quad . \tag{5-8}$$

Substituting for the quantities $\frac{\partial U}{\partial x_0}$, $\frac{\partial U}{\partial x_i}$, and $\frac{\partial U}{\partial t_i} \frac{\partial t_i}{\partial z_i}$ from Eqs. (3-12), (3-13), and (3-19), respectively, we obtain

$$\frac{\partial U}{\partial D_0} = v_z W_0 \frac{\partial x_0}{\partial D_0} , \tag{5-9}$$

$$\frac{\partial U}{\partial \alpha_0} = v_z W_0 \frac{\partial x_0}{\partial \alpha_0} , \tag{5-10}$$

$$\frac{\partial U}{\partial z_i} = v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} , \tag{5-11}$$

$$\frac{\partial U}{\partial \varepsilon_i} = v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} , \tag{5-12}$$

and

$$\frac{\partial U}{\partial \alpha_i} = v_z W_i \frac{\partial x_i}{\partial \alpha_i} , \tag{5-13}$$

where v_z is the balloon ascent rate, and W_0, W_i, and W_{ui} are defined by Eqs. (3-9), (3-10), and (3-20), respectively.

Analogous expressions for the North component of ballistic wind can be obtained in the same fashion. For example,

$$\frac{\partial V}{\partial z_i} = v_z W_i \frac{\partial y_i}{\partial z_i} + W_{vi} , \tag{5-14}$$

where W_{vi} is defined by Eq. (3-24).

When the values of the partial derivatives discussed above are appropriately substituted into Eq. (5-2) or Eq. (5-3), the resulting expressions are rather unwieldy. In order to formulate the results, we adopt a mnemonic code to represent each of the 18 error sums in Eqs. (5-2) and (5-3). (The first term

in brackets in each equation is taken to be a single error sum.) This code is displayed in Table I. Code names beginning with the letter B represent bias sums before squaring and thus have units of velocity; code names beginning with the letter R represent sums of the squares of random error contributions and thus have units of velocity squared.

The error sums, after substitution for the partial derivatives of U and V, are displayed in Tables II and III. The forms of the partial derivatives of the geometric variables, e.g., $\frac{\partial x_i}{\partial z_i}$, $\frac{\partial x_i}{\partial \varepsilon_i}$ etc., are given in Appendix B.

In RAWIN (and also in the RADAR model), it is assumed that single values are valid over all zones for the following: bias error $\sigma_{B\varepsilon L}$ in elevation due to causes other than ground reflection; random error $\sigma_{R\varepsilon L}$ in elevation due to causes other than ground reflection; bias error $\sigma_{B\alpha}$ in azimuth; and random error $\sigma_{R\alpha}$ in azimuth. The remaining errors may vary from zone to zone and hence retain the subscript i in Tables II and III.

Each of the errors $\sigma_{B\varepsilon iG}$ and $\sigma_{R\varepsilon iG}$ due to ground reflection depends on (among other things) the angle of incidence that the incoming signal makes with respect to the surface of the earth. If the terrain in front of the receiving set is sloping, the angle of incidence will be affected. This should be taken into account in calculating the sums BERXE, RERXE, BERYE, and RERYE. In RAWIN (and also in RADAR), a positive or zero angle of slope, called the foreground elevation F, is required as input. The model utilizes F to determine the correct angle of incidence. It then uses linear interpolation to select proper values of $\sigma_{B\varepsilon iG}$ and $\sigma_{R\varepsilon iG}$ from inputted tables. These tables must contain externally generated values of $\sigma_{B\varepsilon jG}$ and $\sigma_{R\varepsilon jG}$, $j = 1, 2, \ldots, 271$, corresponding to potential elevation angles of 0.0, 0.33, 0.67, 1.0, 1.33, ..., 90.0 degrees, respectively.

Table IV summarizes all the inputs required by the RAWIN model for the case of a complete zone structure containing N_Z zones. Conversion of input units to MKS values is done by the model itself where necessary.

The variance in the East component of ballistic wind is given by

$$\sigma_U^2 = REXL + (BEXZ)^2 + (BELXE)^2 + (BERXE)^2 + (BEXA)^2 + REXZ + RELXE + RERXE + REXA. \tag{5-15}$$

while the variance in the North component is given by

$$\sigma_V^2 = REYL + (BEYZ)^2 + (BELYE)^2 + (BERYE)^2 + (BEYA)^2 + REYZ + RELYE + RERYE + REYA. \tag{5-16}$$

The CVV is computed from Eq. (4-13). On output the units are converted to knots squared.

Table I. Mnemonic Code for RAWIN Error Sums

Error Source	East		North	
	Bias (m/sec)	Random $(m/sec)^2$	Bias (m/sec)	Random $(m/sec)^2$
Launch Position Determination	--	REXL	--	REYL
Altitude Determination	BEXZ	REXZ	BEYZ	REYZ
Elevation Measurement ("Leveling")	BELXE	RELXE	BELYE	RELYE
Elevation Measurement (Reflection)	BERXE	RERXE	BERYE	RERYE
Azimuth Measurement	BEXA	REXA	BEYA	REYA

Table II. RAWIN Error Sums for the East Component of Ballistic Wind (Line n)

$$\text{REXL} = \left[v_z W_0 \frac{\partial x_0}{\partial D_0} \right]^2 \sigma_{RD0}^2 + \left[v_z W_0 \frac{\partial x_0}{\partial \alpha_0} \right]^2 \sigma_{R\alpha 0}^2$$

$$\text{BEXZ} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right] \sigma_{Bzi}$$

$$\text{REXZ} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right]^2 \sigma_{Rzi}^2$$

$$\text{BELXE} = \sum_{i=1}^{n} v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \sigma_{B\varepsilon L}$$

$$\text{RELXE} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right]^2 \sigma_{R\varepsilon L}^2$$

$$\text{BERXE} = \sum_{i=1}^{n} v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \sigma_{B\varepsilon iG}$$

$$\text{RERXE} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right]^2 \sigma_{R\varepsilon iG}^2$$

$$\text{BEXA} = \sum_{i=1}^{n} v_z W_i \frac{\partial x_i}{\partial \alpha_i} \sigma_{B\alpha}$$

$$\text{REXA} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial \alpha_i} \right]^2 \sigma_{R\alpha}^2$$

Table III. RAWIN Error Sums for the North Component of Ballistic Wind (Line n)

$$\text{REYL} = \left[v_z W_0 \frac{\partial y_0}{\partial D_0} \right]^2 \sigma^2_{RD0} + \left[v_z W_0 \frac{\partial y_0}{\partial \alpha_0} \right]^2 \sigma^2_{R\alpha 0}$$

$$\text{BEYZ} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial y_i}{\partial z_i} + W_{vi} \right] \sigma_{Bzi}$$

$$\text{REYZ} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial y_i}{\partial z_i} + W_{vi} \right]^2 \sigma^2_{Rzi}$$

$$\text{BELYE} = \sum_{i=1}^{n} v_z W_i \frac{\partial y_i}{\partial \varepsilon_i} \sigma_{B\varepsilon L}$$

$$\text{RELYE} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial y_i}{\partial \varepsilon_i} \right]^2 \sigma^2_{R\varepsilon L}$$

$$\text{BERYE} = \sum_{i=1}^{n} v_z W_i \frac{\partial y_i}{\partial \varepsilon_i} \sigma_{B\varepsilon iG}$$

$$\text{RERYE} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial y_i}{\partial \varepsilon_i} \right]^2 \sigma^2_{R\varepsilon iG}$$

$$\text{BEYA} = \sum_{i=1}^{n} v_z W_i \frac{\partial y_i}{\partial \alpha_i} \sigma_{B\alpha}$$

$$\text{REYA} = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial y_i}{\partial \alpha_i} \right]^2 \sigma^2_{R\alpha}$$

Table IV. RAWIN Inputs

Symbol	Input Units	No. of Values	Explanation
D_0	m	1	Launch Displ.: used to compute partials
F	deg	1	Foreground Elev.: used in selecting values of $\sigma_{B\varepsilon iG}$ and $\sigma_{R\varepsilon iG}$ from $\sigma_{B\varepsilon jG}$ and $\sigma_{R\varepsilon jG}$
u_i	m/sec	N_z	East zone wind vel.: used to compute W_{ui}
v_i	m/sec	N_z	North zone wind vel.: used to compute W_{vi}
v_z	m/min	1 to 4	Balloon ascent rate
w_{ik}		N_z^2	Complete table of wind weighting factors, where $i = 1, \ldots, N_z$ and $k = 1, \ldots, N_z$; for any line n, the model selects approp. values of w_i which are used to compute W_0, W_i, W_{ui}, W_{vi}.
z_i	m	N_z	Zone top alt.: used to compute W_0, W_i, W_{ui}, W_{vi}
α_0	deg	1	Launch azimuth: used to compute partials
α_i	deg	N_z	Zone top azimuth: Used to compute partials
ε_i	deg	N_z	Zone top elev.: used to compute partials
σ_{RD0}	m	1	Random error in launch displacement
$\sigma_{R\alpha 0}$	deg	1	Random error in launch azimuth
σ_{Bzi}	m	N_z	Bias error in altitude
σ_{Rzi}	m	N_z	Random error in altitude
$\sigma_{B\alpha}$	deg	1	Bias error in azim. tracking of apparent target
$\sigma_{R\alpha}$	deg	1	Random error in azim. tracking of apparent target
$\sigma_{B\varepsilon L}$	deg	1	Bias error in elev. tracking of apparent target
$\sigma_{R\varepsilon L}$	deg	1	Random error in elev. tracking of apparent target
$\sigma_{B\varepsilon jG}$	deg	271	Potential bias errors in elev. due to ground reflection: used to determine $\sigma_{B\varepsilon iG}$
$\sigma_{R\varepsilon jG}$	deg	271	Potential Random errors in elev. due to ground reflection: used to determine $\sigma_{R\varepsilon iG}$

NOTE: N_z = 15 for the NATO zone structure.

6. RADAR

RADAR models the case in which a ground-based radar set measures the independent variables of slant range, elevation angle, and azimuth associated with the radiosonde's position. This model is applicable, for example, to the Radar Sets AN/TMQ-19 and AN/FPS-16.

The derivation of the variance in each component of ballistic wind follows essentially the same path as in RAWIN. The main difference in RADAR is that the altitude z_i at zone top i is not considered to be an independent variable but rather is a function of slant range S_i and the elevation angle ε_i. Then from Eq. (5-1), we may indicate the functional form of the East component U of ballistic wind by

$$U = U\,[x_0\,(D_0,\,\alpha_0),\,t_0,\,x_i\,(z_i\,(S_i,\,\varepsilon_i),\,\varepsilon_i,\,\alpha_i),\,t_i\,(z_i\,(S_i,\,\varepsilon_i))], \tag{6-1}$$

$$i = 1,\,2,\,\ldots,\,n\quad,$$

where all symbols retain their previously defined meanings.

It is, of course, not necessary to write U in precisely the form shown in Eq. (6-1). We could, for example, omit the intermediate variable z_i entirely. However, Eq. (6-1) permits us to make further use in radar of several partial derivatives that are also valid in RAWIN.

The variance $\sigma_U{}^2$ in U is obtained by specializing Eq. (4-3) to the independent variables D_0, α_0, S_i, ε_i, and α_i. The result is

$$\sigma_U{}^2 = [(\frac{\partial U}{\partial D_0}\,\sigma_{RD0})^2 + (\frac{\partial U}{\partial \alpha_0}\,\sigma_{R\alpha 0})^2] +$$

$$+ \left[\sum_{i=1}^{n} \frac{\partial U}{\partial S_i} \sigma_{BSi} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \varepsilon_i} \sigma_{B\varepsilon iL} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \varepsilon_i} \sigma_{B\varepsilon iG} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \alpha_i} \sigma_{B\alpha i} \right]^2 \tag{6-2}$$

$$+ \sum_{i=1}^{n} \left[\frac{\partial U}{\partial S_i} \sigma_{RSi}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \varepsilon_i} \sigma_{R\varepsilon iL}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \varepsilon_i} \sigma_{R\varepsilon iG}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \alpha_i} \sigma_{R\alpha i}\right]^2 .$$

In Eq. (6-2), each type of error is designated by σ with appropriate subscripts. The meanings of the subscripts are identified by the rules that are stated below Eq. (5-2). For example, σ_{RSi} is the random error in the measurement of slant range for the i'th zone top. As before, we assume that any error in the launch time is negligible and omit a term involving t_0 in Eq. (6-2).

Similarly, the variance σ_V^2 in th· North component V of ballistic wind is

$$\sigma_V^2 = \left[\left(\frac{\partial V}{\partial D_0} \sigma_{RD0}\right)^2 + \left(\frac{\partial V}{\partial \alpha_0} \sigma_{R\alpha 0}\right)^2 \right]$$

$$+ \left[\sum_{i=1}^{n} \frac{\partial V}{\partial S_i} \sigma_{BSi} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial V}{\partial \varepsilon_i} \sigma_{B\varepsilon iL} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial V}{\partial \varepsilon_i} \sigma_{B\varepsilon iG} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial V}{\partial \alpha_i} \sigma_{B\alpha i} \right]^2 \tag{6-3}$$

$$+ \sum_{i=1}^{n} \left[\frac{\partial V}{\partial S_i} \sigma_{RSi}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial V}{\partial \varepsilon_i} \sigma_{R\varepsilon iL}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial V}{\partial \varepsilon_i} \sigma_{R\varepsilon iG}\right]^2 + \sum_{i=1}^{n} \left[\frac{\partial V}{\partial \alpha_i} \sigma_{R\alpha i}\right]^2 .$$

We use the chain rule for partial derivatives in conjunction with Eq. (6-1) to obtain

$$\frac{\partial U}{\partial D_0} = \frac{\partial U}{\partial x_0} \frac{\partial x_0}{\partial D_0} , \tag{6-4}$$

$$\frac{\partial U}{\partial \alpha_0} = \frac{\partial U}{\partial x_0} \frac{\partial x_0}{\partial \alpha_0} , \tag{6-5}$$

$$\frac{\partial U}{\partial S_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial z_i} \frac{\partial z_i}{\partial S_i} + \frac{\partial U}{\partial t_i} \frac{\partial t_i}{\partial z_i} \frac{\partial z_i}{\partial S_i} , \tag{6-6}$$

$$\frac{\partial U}{\partial \varepsilon_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial z_i} \frac{\partial z_i}{\partial \varepsilon_i} + \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial \varepsilon_i} + \frac{\partial U}{\partial t_i} \frac{\partial t_i}{\partial z_i} \frac{\partial z_i}{\partial \varepsilon_i} , \tag{6-7}$$

and

$$\frac{\partial U}{\partial \alpha_i} = \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} . \tag{6-8}$$

With appropriate substitution from Eqs. (3-12), (3-13), and (3-19), the above expressions become

$$\frac{\partial U}{\partial D_0} = v_z W_0 \frac{\partial x_0}{\partial D_0} , \tag{6-9}$$

$$\frac{\partial U}{\partial \alpha_0} = v_z W_0 \frac{\partial x_0}{\partial \alpha_0} , \tag{6-10}$$

$$\frac{\partial U}{\partial S_i} = (v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui}) \frac{\partial z_i}{\partial S_i} , \tag{6-11}$$

$$\frac{\partial U}{\partial \varepsilon_i} = (v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui}) \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} , \tag{6-12}$$

and

$$\frac{\partial U}{\partial \sigma_i} = v_z W_i \frac{\partial x_i}{\partial \alpha_i} , \tag{6-13}$$

where v_z is the balloon ascent rate, and W_0, W_i, and W_{ui} are defined by Eqs. (3-9), (3-10), and (3-20), respectively.

Analogous expressions involving the North component of ballistic wind can be obtained. For example,

$$\frac{\partial V}{\partial S_i} = (v_z W_i \frac{\partial y_i}{\partial z_i} + W_{vi}) \frac{\partial z_i}{\partial S_i} \quad , \tag{6-14}$$

where W_{vi} is defined by Eq. (3-24).

The 18 error sums in Eqs. (6-2) and (6-3) are represented by the mnemonic code presented in Table V. The algebraic forms of these sums, after substitution for the various partial derivatives of U and V, are given in Tables VI and VII.

Since the altitudes z_i define the zone structure, they are still required as inputs to RADAR. Conversely, actual values of slant range S_i are not required. Such values could be used to compute some of the partial derivatives shown in Tables VI and VII. However, values of z_i also serve in this capacity and are used instead. See Appendix B.

The discussion near the end of Section 5 concerning errors in azimuth, errors in elevation, and the foreground elevation is also pertinent to the RADAR model. In addition, RADAR requires a single inputted value, valid for all zones, for the random error σ_{RS} in slant range and also a single value for the bias error σ_{BS}. Although it might be expected that σ_{BS} would be negligible, it is nevertheless included here for generality.

Table VIII summarizes all the inputs required by the RADAR model for the case of a complete zone structure containing N_z zones. Conversion of input units to MKS values is done by the model itself where necessary.

The variance in the East component of ballistic wind is given by

$$\sigma_U^2 = REXL + (BEXS)^2 + (BELXE)^2 + (BERXE)^2 + (BEXA)^2 + REXS + RELXE + RERXE + REXA \quad , \tag{6-15}$$

while the variance in the North component is

$$\sigma_V^2 = REYL + (BEYS)^2 + (BELYE)^2 + (BERYE)^2 + (BEYA)^2 + REYS + RELYE + RERYE + REYA \tag{6-16}$$

The CVV is computed from Eq. (4-13). Output is in knots squared.

Table V. Mnemonic Code for *RADAR* Error *Sums*

Error Source	East		North	
	Bias (m/sec)	Random $(m/sec)^2$	Bias (m/sec)	Random $(m/sec)^2$
Launch Position Determination	--	REXL	--	REYL
Slant Range Measurement	BEXS	REXS	BEYS	REYS
Elevation Measurement ("Leveling")	BELXE	RELXE	BELYE	RELYE
Elevation Measurement (Reflection)	BERXE	RERXE	BERYE	RERYE
Azimuth Measurement	BEXA	REXA	BEYA	REYA

Table VI. RADAR Error Sums for the East Component of Ballistic Wind (Line n)

$$REXL = \left[v_z W_0 \frac{\partial x_0}{\partial D_0} \right]^2 \sigma^2_{RD0} + \left[v_z W_0 \frac{\partial x_0}{\partial \alpha_0} \right]^2 \sigma^2_{R\alpha 0}$$

$$BEXS = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right) \frac{\partial z_i}{\partial S_i} \right] \sigma_{BS}$$

$$REXS = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right) \frac{\partial z_i}{\partial S_i} \right]^2 \sigma^2_{RS}$$

$$BELXE = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right) \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right] \sigma_{B\varepsilon L}$$

$$RELXE = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right) \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right]^2 \sigma^2_{R\varepsilon L}$$

$$BERXE = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial z_i} + W_{ui} \right) \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right] \sigma_{B\varepsilon iG}$$

$$RERXE = \sum_{i=1}^{n} \left[\left(v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} + W_{ui} \right) \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \frac{\partial x_i}{\partial \varepsilon_i} \right]^2 \sigma^2_{R\varepsilon iG}$$

$$BEXA = \sum_{i=1}^{n} v_z W_i \frac{\partial x_i}{\partial \alpha_i} \sigma_{B\alpha}$$

$$REXA = \sum_{i=1}^{n} \left[v_z W_i \frac{\partial x_i}{\partial \alpha_i} \right]^2 \sigma^2_{R\alpha}$$

Table VII. RADAR Error Sums for the North Component of Ballistic Wind (Line n)

$$REYL = [\ v_z W_0 \frac{\partial y_0}{\partial D_0}]^2 \ \sigma^2_{RD0} + [\ v_z W_0 \ \frac{\partial y_0}{\partial \alpha_0}]^2 \ \sigma^2_{R\alpha 0}$$

$$BEYS = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial z_i} + W_{vi}) \ \frac{\partial z_i}{\partial S_i}]^2 \ \sigma_{BS}$$

$$REYS = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial z_i} + W_{vi}) \ \frac{\partial z_i}{\partial S_i}]^2 \ \sigma^2_{RS}$$

$$BELYE = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial z_i} + W_{vi}) \ \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \ \frac{\partial y_i}{\partial \varepsilon_i}] \ \sigma_{B\varepsilon L}$$

$$RELYE = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial z_i} + W_{vi}) \ \frac{\partial z_i}{\partial \varepsilon} + v_z W_i \ \frac{\partial y_i}{\partial \varepsilon_i}]^2 \ \sigma^2_{R\varepsilon L}$$

$$BERYE = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial z_i} + W_{vi}) \ \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \ \frac{\partial y_i}{\partial \varepsilon_i}] \ \sigma_{B\varepsilon iG}$$

$$RERYE = \sum_{i=1}^{n} [(\ v_z W_i \ \frac{\partial y_i}{\partial \varepsilon_i} + W_{vi}) \ \frac{\partial z_i}{\partial \varepsilon_i} + v_z W_i \ \frac{\partial y_i}{\partial \varepsilon_i}]^2 \ \sigma^2_{R\varepsilon iG}$$

$$BEYA = \sum_{i=1}^{n} v_z W_i \ \frac{\partial y_i}{\partial \alpha_i} \ \sigma_{B\alpha}$$

$$REYA = \sum_{i=1}^{n} [\ v_z W_i \ \frac{\partial y_i}{\partial \alpha_i}]^2 \ \sigma^2_{R\alpha}$$

Table VIII. RADAR Inputs

Symbol	Input Units	No. of Values	Explanation
D_0	m	1	Launch Displ.: used to compute partials
F	deg	1	Foreground Elev.: used in selecting values of $\sigma_{B\varepsilon iG}$ and $\sigma_{R\varepsilon iG}$ from $\sigma_{B\varepsilon jG}$ and $\sigma_{R\varepsilon jG}$
u_i	m/sec	N_z	East zone wind vel.: used to compute W_{ui}
v_i	m/sec	N_z	North zone wind vel.: used to compute W_{vi}
v_z	m/min	1 to 4	Balloon ascent rate
w_{ik}		N_z^2	Complete table of wind weighting factors, where $i = 1, \ldots, N_z$ and $k = 1, \ldots, N_z$; for any line n, the model selects approp. values of w_i which are used to compute W_0, W_i, W_{ui}, W_{vi}.
z_i	m	N_z	Zone top alt.: used to compute W_0, W_i, W_{ui}, W_{vi}
α_0	deg	1	Launch azimuth: used to compute partials
α_i	deg	N_z	Zone top azimuth: used to compute partials
ε_i	deg	N_z	Zone top elev.: used to compute partials
σ_{RD0}	m	1	Random error in launch displacement
$\sigma_{R\alpha 0}$	deg	1	Random error in launch azimuth
σ_{BS}	m	1	Bias error in slant range
σ_{RS}	m	1	Random error in slant range
$\sigma_{B\alpha}$	deg	1	Bias error in azim. tracking of apparent target
$\sigma_{R\alpha}$	deg	1	Random error in azim. tracking of apparent target
$\sigma_{B\varepsilon L}$	deg	1	Bias error in elev. tracking of apparent target
$\sigma_{R\varepsilon L}$	deg	1	Random error in elev. tracking of apparent target
$\sigma_{B\varepsilon jG}$	deg	271	Potential bias errors in elev. due to ground reflection: used to determine $\sigma_{B\varepsilon iG}$
$\sigma_{R\varepsilon jG}$	deg	271	Potential Random errors in elev. due to ground reflection: used to determine $\sigma_{R\varepsilon iG}$

NOTE: N_z = 15 for the NATO zone structure.

7. NAVAID

NAVAID models the case in which radionavigation techniques are used to determine the position of the radiosonde. See Reference 3, for example, for a review of these techniques. NAVAID is a simplified model in that it bypasses the complexities of hyperbolic geometry by requiring estimated errors in the East and North components of position as input. This model is applicable to the radionavigation portions (LORAN, VLF, and OMEGA) of FAMAS.

To obtain the implicit functional form of the East component U of ballistic wind appropriate to line n, we rewrite Eq. (4-1) in the following way:

$$U = U\,[x_0,\ t_0,\ x_i,\ t_i\ (z_i)],\ i = 1,\ 2,\ \ldots,\ n. \tag{7-1}$$

The independent variables are taken to be the East launch coordinate x_0, the launch time t_0, the radiosonde's East coordinate x_i at the top of zone i, and the corresponding altitude z_i. As in RAWIN, the time t_i is considered to be a function of z_i.

In hyperbolic tracking any bias errors in x_i and in y_i, the North component of position, are expected to be negligible. Therefore, bias error sums are not computed for these variables in NAVAID. However, we do allow for both bias and random errors in the determination of z_i. As in the other models, it is assumed that there is no error associated with t_0.

Specializing Eq. (4-3) to NAVAID, we have for the variance $\sigma_U{}^2$ in U:

$$\begin{aligned}\sigma_U{}^2 = {} & \left[\frac{\partial U}{\partial x_0}\ \sigma_{Rx0} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial z_i}\ \sigma_{Bzi} \right]^2 \\ & + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial x_i}\ \sigma_{Rxi} \right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial z_i}\ \sigma_{Rzi} \right]^2 \quad , \end{aligned} \tag{7-2}$$

where each type of error is designated by σ with appropriate subscripts. The meanings of the subscripts are identified by the rules stated below Eq. (5-2). For example, σ_{Rxi} is the random error in the East coordinate for zone top i. A similar expression holds for the variance $\sigma_V{}^2$ in the North component V of ballistic wind:

$$\sigma_V{}^2 = [\frac{\partial V}{\partial y_0} \sigma_{Ry0}]^2 + [\sum_{i=1}^{n} \frac{\partial V}{\partial z_i} \sigma_{Bzi}]^2$$

$$+ \sum_{i=1}^{n} [\frac{\partial V}{\partial y_i} \sigma_{Ryi}]^2 + \sum_{i=1}^{n} [\frac{\partial V}{\partial z_i} \sigma_{Rzi}]^2 \quad . \tag{7-3}$$

The mnemonic code representing the eight error sums in Eqs. (7-2) and (7-3) is given in Table IX.

The partial derivatives $\frac{\partial U}{\partial x_0}$, $\frac{\partial U}{\partial x_i}$, $\frac{\partial V}{\partial y_0}$, and $\frac{\partial V}{\partial y_i}$ are given by Eqs. (3-12), (3-13), (3-21), and (3-22), respectively. Using the chain rule for partial derivatives and substituting from Eqs. (3-19) and (3-23), we also have

$$\frac{\partial U}{\partial z_i} = W_{ui} \tag{7-4}$$

and

$$\frac{\partial V}{\partial z_i} = W_{vi} \quad , \tag{7-5}$$

where W_{ui} and W_{vi} are defined by Eqs. (3-20) and (3-24), respectively.

NAVAID computes the random positional errors σ_{Rxi} and σ_{Ryi} using th procedure outlined below.

It is assumed that the hyperbolic tracking system fixes, i.e., determines, the East and North components of the radiosonde's position every $\Delta\tau$ seconds. We take the random error in the fixing of each component to be a constant that is characteristic of the measuring system. These errors, labeled $\sigma_{Rx\tau}$ and $\sigma_{Ry\tau}$, respectively, are required as inputs to NAVAID.

Consider a height interval of ΔH_i meters, which is centered at the top of zone i. As the radiosonde traverses ΔH_i, N_i fixes of position are made. We have

$$N_i = \text{Integer} \left[\frac{\Delta H_i}{v_z \Delta\tau} \right] , \tag{7-6}$$

where the ascent rate v_z is expressed in meters per second.

The value of x_i at the zone top is taken to be the mean of the N_i fixes of the radiosonde's East coordinate. Under this condition σ^2_{Rxi} is given by

$$\sigma^2_{Rxi} = \frac{\sigma^2_{Rx\tau}}{N_i} . \tag{7-7}$$

Similarly,

$$\sigma^2_{Ryi} = \frac{\sigma^2_{Ry\tau}}{N_i} . \tag{7-8}$$

The component launch errors σ_{Rx0} and σ_{Ry0} are handled in the following way. As the radiosonde sits on the ground, its position can be determined by hyperbolic fixing during some time interval. The radiosonde-balloon combination may then be transported to another nearby position for actual launch. The second position is determined relative to the first by some direct method. Since the errors in these two different types of measurements are uncorrelated, we have for the East component:

$$\sigma^2_{Rx0} = \sigma^2_{Rx0F} + \sigma^2_{Rx0D} \quad , \tag{7-9}$$

where σ_{Rx0F} is the component random error in the hyperbolic fixing of the first position, and σ_{Rx0D} is the component random error in the direct determination of the second position relative to the first.

If the fixing of the initial position occurs over a time interval of ΔT minutes, then the number N_0 of fixes is given by

$$N_0 = \text{Integer } [60 \frac{\Delta T}{\Delta \tau}] \quad , \tag{7-10}$$

and the variance is

$$\sigma^2_{Rx0F} = \frac{\sigma^2_{Rx\tau}}{N_0} \quad . \tag{7-11}$$

We have no foreknowledge of the value of σ_{Rx0D} nor of the corresponding North error σ_{Ry0D}. For any particular launch they are not necessarily equal to each other. However, on average over many launches, we expect the following relationship:

$$\sigma^2_{Rx0D} = \sigma^2_{Ry0D} = \frac{\sigma^2_{RD0}}{2} \quad , \tag{7-12}$$

where σ_{RD0} is the random error in the direct measurement of the distance from the initial position to the final launch position. For the purpose of modeling the data acquisition system, Eq. (7-12) is assumed to hold. The quantity σ_{RD0} is referred to as the random launch error and is a required input to NAVAID.

From Eqs. (7-9), (7-11), and (7-12), we have

$$\sigma_{Rx0}^{2} = \frac{\sigma_{Rx\tau}^{2}}{N_0} + \frac{\sigma_{RD0}^{2}}{2} \quad . \tag{7-13}$$

An analogous equation yields σ_{Ry0}^{2}.

In order to determine N_i and N_0, NAVAID takes $\Delta\tau$ = 1 second and ΔT = 5 minutes. It also assumes the following arbitrary values: ΔH_i = 200 meters for the first five zones; ΔH_i = 400 meters for the remaining zones. The motivation for this is the NATO zone structure in which the higher zones are significantly thicker than the lower zones. The Users' Manual explains how to alter the values of $\Delta\tau$, ΔT, and ΔH_i.

The preceding analysis is used to find the final form of each of the required bias and random sums. These are displayed in Table X.

The inputs to NAVAID are summarized in Table XI. Conversion of input units to MKS values is done by the model itself where necessary.

The variances in the East and North components, respectively, of ballistic wind are given by

$$\sigma_U^2 = \text{REXL} + (\text{BEXZ})^2 + \text{REXX} + \text{REXZ} \tag{7-14}$$

and

$$\sigma_V^2 = \text{REYL} + (\text{BEYZ})^2 + \text{REYY} + \text{REYZ} \quad . \tag{7-15}$$

The CVV is computed from Eq. (4-13). Output is in knots squared.

Table IX. Mnemonic Code for NAVAID Error Sums

Error Source	East: Bias (m/sec)	East: Random $(m/sec)^2$	North: Bias (m/sec)	North: Random $(m/sec)^2$
Launch Position Determination	--	REXL	--	REYL
Altitude Determination	BEXZ	REXZ	BEYZ	REYZ
Position Fixing	--	REXX	--	REYY

Table X. NAVAID Error Sums (Line n)

$$REXL = v_z^2 \, W_0^2 \left[\frac{\sigma^2_{Rx\tau}}{N_0} + \frac{\sigma^2_{RD0}}{2} \right]$$

$$BEXZ = \sum_{i=1}^{n} W_{ui} \, \sigma_{Bzi}$$

$$REXZ = \sum_{i=1}^{n} W_{ui}^2 \, \sigma^2_{Rzi}$$

$$REXX = \sum_{i=1}^{n} v_z^2 \, W_i^2 \, \frac{\sigma^2_{Rx\tau}}{N_i}$$

$$REYL = v_z^2 \, W_0^2 \left[\frac{\sigma^2_{Ry\tau}}{N_0} + \frac{\sigma^2_{RD0}}{2} \right]$$

$$BEYZ = \sum_{i=1}^{n} W_{vi} \, \sigma_{Bzi}$$

$$REYZ = \sum_{i=1}^{n} W_{vi} \, \sigma^2_{Bzi}$$

$$REYY = \sum_{i=1}^{n} v_z^2 \, W_i^2 \, \frac{\sigma^2_{Ry\tau}}{N_i}$$

Table XI. NAVAID Inputs

Symbol	Input Units	No. of Values	Explanation
u_i	m/sec	N_z	East zone wind vel.: used to compute W_{ui}
v_i	m/sec	N_z	North zone wind vel.: used to compute W_{vi}
v_z	m/min	1 to 4	Balloon ascent rate
w_{ik}		N_z	Complete table of wind weighting factors, where $i = 1, \ldots, N_z$ and $k = 1, \ldots, N_z$; for any line n, the model selects approp. values of w_i, which are used to compute W_0, W_i, W_{ui}, W_{vi}.
z_i	m	N_z	Zone top alt.: used to compute W_0, W_i, W_{ui}, W_{vi}
σ_{RD0}	m	1	Random error in direct determination of launch position
σ_{Bzi}	m	N_z	Bias error in altitude
σ_{Rzi}	m	N_z	Random error in altitude
$\sigma_{Rx\tau}$	m	1	Random error associated with a single hyperbolic fix of the East coordinate
$\sigma_{Ry\tau}$	m	1	Random error associated with a single hyperbolic fix of the North coordinate

NOTE: N_z = 15 for the NATO zone structure.

8. COMMENTS AND SUGGESTIONS FOR FURTHER WORK

In RAWIN and RADAR, errors in elevation due to ground reflection can make significant contributions to the error in ballistic wind, particularly for the case of low elevation angles. These errors depend on a number of factors, including elevation angle, surface dielectric constant, signal frequency, antenna voltage pattern, and the tracking method used. General information on two tracking methods, *sequential lobing and conical scan*, can be found in Reference 4. A limited auxiliary model LRDC exists for computing theoretical values of bias and random reflection errors for an antenna pattern that is indentical for the high and low switched positions; its implementation is described in the Users' Manual.

The bias reflection error generated by LRDC for any given elevation angle does not represent a standard deviation characteristic of a normal population with zero mean error. Consequently, if LRDC is used to generate the required bias reflection inputs to RAWIN and RADAR, it should be recognized that the quantities $\sigma_U{}^2$ and $\sigma_V{}^2$ discussed in this report no longer correspond precisely to variances. (The bias errors computed by LRDC are typically much smaller than the corresponding random reflection errors, which are also computed.)

Most of the error sums obtained in this report for ballistic line n contain the factor W_i, $i = 1, 2, \ldots, n$. As defined by Eq. (3-10), the quantity W_i represents the wind weighting factor per unit zone width for zone i minus the wind weighting factor per unit zone width for zone i + 1. Depending on the weighting factors and zone widths involved, W_i can be positive, negative, or zero. For the case of Message 3 weighting factors, W_n has the largest magnitude of any member of the set $\{W_i,\ i = 1, 2, \ldots, n\}$ for the following reasons: the zone wind weighting factor is greatest for zone n; the wind weighting factor for zone n + 1 is zero. For example, using the NATO zone structure and the corresponding Message 3 wind weighting factors given in Reference 2, one obtains the following values (in meter^{-1}) relevant to ballistic line 7: $W_1 = 0.$; $W_2 = -0.40 \times 10^{-4}$; $W_3 = 0.$; $W_4 = -0.20 \times 10^{-4}$; $W_5 = -0.40 \times 10^{-4}$; $W_6 = -0.33 \times 10^{-3}$; $W_7 = 0.53 \times 10^{-3}$. The way in which the W_i are calculated results in a "cancellation effect" for i less than n. In fact, if $W_i = 0.$, the contribution of zone i to some of the error sums will be

nullified. This is not necessarily what one would expect intuitively. In order to verify whether or not the cancellation effect is real, appropriate comparisons between model predictions and experimental results should be attempted.

Each of the models utilizes a single zone wind profile, u_i, v_i, $i = 1, 2, \ldots, N_z$ (where n_z is the total number of zones in the structure), valid for all balloon ascent rates. From a known or postulated profile, tables of values of elevation ε_i and α_i, $i = 1, 2, \ldots, N_z$ can be developed fo. each balloon ascent rate for use in RAWIN or RADAR. Alternatively, if the values of ε_i and α_i are known, a zone wind profile can be calculated. In either case, the tables of values must be obtained externally to the model and used as input to it. Although the procedure for doing this is conceptually straightforward, it would be useful to incorporate it into the models themselves or, alternatively, create an auxiliary model that would generate the necessary tables.

The zone wind profile is, of course, merely a useful representation of actual wind conditions aloft. Real winds do not necessarily maintain constant magnitude and direction over an arbitrary zone width and then abruptly assume new values at a zone boundary. Thus the zone wind methodology introduces an artificial discontinuity or "shear" in wind at the zone boundaries. Any error in the determination of the zone top altitudes will lead to a calculated zone wind profile that does not correspond precisely to the zone structure under study. The form of the quantities W_{ui} and W_{vi} in Eqs. (3-20) and (3-24), respectively, and the manner in which they were obtained suggest that they are related to first-order wind shear contributions to the error in ballistic wind. Further work on this point and on the entire question of wind shear is suggested.

The models presented here represent one approach to the subject of ballistic wind measurement error analysis. They are not, of course, the final word on this subject. The subsequent use of these models should help to establish their strong points, as well as locate areas in need of improvement or extension.

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APPENDIX A. VARIANCE ASSOCIATED WITH A COMPONENT OF BALLISTIC WIND

Using appropriate assumptions, we want to show that Eq. (4-3) yields the variance in the East component U of ballistic wind for line n.

The generalized implicit functional form of U is given by Eq. (4-2):

$$U = U\,[\lambda_{10}, \lambda_{20}, \ldots, \lambda_{L0}, \xi_{1i}, \xi_{2i}, \ldots, \xi_{ki}],\ i=1, 2, \ldots, n. \tag{A-1}$$

The subsequent analysis is simplified somewhat if the number of subscripts in Eq. (A-1) is reduced. To this end we rewrite the subset of independent launch variables λ_{10}, λ_{20} ..., λ_{L0}, as μ_0, ν_0, ..., and we rewrite the subset of independent variables ξ_{1i}, ξ_{2i}, ..., ξ_{ki}, determined for the top of zone i, as ρ_i, ζ_i, ..., explicitly retaining only two members of each subset. Then,

$$U = U\,[\mu_0, \nu_0, \ldots, \rho_i, \zeta_i, \ldots],\ i = 1, 2, \ldots, n\quad . \tag{A-2}$$

In order to obtain the expression for the variance in U, we consider a Gedanken experiment in which a large number M of balloon flights is carried out under the same meteorological conditions. For the m'th flight the independent variables listed in Eq. (A-2) have the measured values μ_{0m}, ν_{0m}, ..., ρ_{im}, ζ_{im}, ..., respectively. These can be used to calculate a component value U_m of ballistic wind, appropriate to flight m.

We associate U with the mean of the set of values U_m, m = 1, 2, ..., M. Let δU_m be the deviation of U_m from the mean. To first order δU_m is given by the Taylor expansion:

$$\delta U_m = \frac{\partial U}{\partial \mu_0} \delta\mu_{0m} + \frac{\partial U}{\partial v_0} \delta v_{0m} + \ldots + \sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \delta\rho_{im} + \sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \delta\zeta_{im} + \ldots, \tag{A-3}$$

where each of the quantities $\delta\mu_{0m}$, δv_{0m}, ..., $\delta\rho_{im}$, $\delta\zeta_{im}$, ... represents the deviation from the mean in the corresponding variable. The partial derivatives are evaluated using the mean value of each variable appropriate to zone top i or to launch position.

For large M the variance σ_U^2 in U is equivalent to the mean squared deviation:

$$\sigma_U^2 = \frac{1}{M} \sum_{m=1}^{M} (\delta U_m)^2 \quad . \tag{A-4}$$

Thus,

$$\sigma_U^2 = \frac{1}{M} \sum_{m=1}^{M} \cdot \left[\frac{\partial U}{\partial \mu_0} \delta\mu_{0m} + \frac{\partial U}{\partial v_0} \delta v_{0m} + \ldots \right.$$

$$\left. + \sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \delta\rho_{im} + \sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \delta\zeta_{im} + \ldots \right]^2 \quad . \tag{A-5}$$

Expansion of the square in Eq. (A-5) yields

$$\sigma_U^2 = \frac{1}{M} \sum_{m=1}^{M} \left[\frac{\partial U}{\partial \mu_0} \delta\mu_{0m} \right]^2 + \frac{1}{M} \sum_{m=1}^{M} \left[\frac{\partial U}{\partial v_0} \delta v_{0m} \right]^2 + \ldots$$

$$+ \frac{1}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho} \delta\rho_{im} \right]^2 + \frac{1}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \delta\zeta_{im} \right]^2 + \ldots$$

$$+ \text{ CROSS TERMS} \quad . \tag{A-6}$$

One of the cross terms in Eq. (A-6) is given by $C_{\rho\zeta}$, where

$$C_{\rho\zeta} = \frac{2}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \delta\rho_{im} \sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \delta\zeta_{im} \right] . \qquad \text{(A-7)}$$

Expansion of the sums in Eq. (A-7) yields

$$C_{\rho\zeta} = \frac{2}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial U}{\partial \rho_i} \frac{\partial U}{\partial \zeta_j} \delta\rho_{im} \delta\zeta_{jm} \right] . \qquad \text{(A-8)}$$

For this form the sum over m may be performed first:

$$C_{\rho\zeta} = 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial U}{\partial \rho_i} \frac{\partial U}{\partial \zeta_i} \left[\frac{1}{M} \sum_{m=1}^{M} \delta\rho_{im} \delta\zeta_{jm} \right] . \qquad \text{(A-9)}$$

We assume that deviations in ρ_i, $i = 1, 2, \ldots, n$, are not correlated with deviations in ζ_j, $j = 1, 2, \ldots, n$. Then for large M the expression in brackets in Eq. (A-9) is taken to be zero. Hence the cross term $C_{\rho\zeta}$ is zero.

A similar argument can be used to show that all the cross terms in Eq. (A-6) are zero. Thus we can rewrite this equation as

$$\sigma_U{}^2 = T_\mu + T_\nu + \ldots + T_\rho + T_\zeta + \ldots , \qquad \text{(A-10)}$$

where

$$T_\mu = \left(\frac{\partial U}{\partial \mu_0}\right)^2 \frac{1}{M} \sum_{m=1}^{M} (\delta\mu_{0m})^2 , \qquad \text{(A-11)}$$

$$T_\nu = \left(\frac{\partial U}{\partial \nu_0}\right)^2 \frac{1}{M} \sum_{m=1}^{M} (\delta\nu_{0m})^2 , \qquad \text{(A-12)}$$

$$T_\rho = \frac{1}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \delta\rho_{im} \right]^2 , \tag{A-13}$$

and

$$T_\zeta = \frac{1}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \delta\zeta_{im} \right]^2 . \tag{A-14}$$

The mean squared deviations in Eqs. (A-11) and (A-12) represent variances. Thus we write

$$T_\mu = \left(\frac{\partial U}{\partial \mu_0}\right)^2 \sigma^2_{R\mu 0} , \tag{A-15}$$

and

$$T_v = \left(\frac{\partial U}{\partial v_0} \right)^2 \sigma^2_{Rv0} , \tag{A-16}$$

where $\sigma^2_{R\mu 0}$ is the variance in μ_0, and σ^2_{Rv0} is the variance in v_0. The subscript R indicates that the launch errors are taken to be random.

We assume that the deviation $\delta\rho_{im}$ from the mean is the sum of a "bias" deviation $\delta\rho_{Bim}$ and a random deviation $\delta\rho_{Rim}$. The $\delta\rho_{Bim}$ are correlated from zone to zone, while the $\delta\rho_{Rim}$ are uncorrelated. Then Eq. (A-13) becomes

$$T_\rho = \frac{1}{M} \sum_{m=1}^{M} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} (\delta\rho_{Bim} + \delta\rho_{Rim}) \right]^2 . \tag{A-17}$$

Upon expansion of the square, Eq. (A-17) becomes

$$T_\rho = \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\frac{\partial U}{\partial \rho_i}\,\delta\rho_{Bim}\right]^2 + \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\frac{\partial U}{\partial \rho_i}\,\delta\rho_{Rim}\right]^2 \quad , \qquad \text{(A-18)}$$

where the omitted cross term is zero because the $\delta\rho_{Rim}$ are not correlated with the $\delta\rho_{Bim}$.

We rewrite Eq. (A-18) as

$$T_\rho = T_{B\rho} + T_{R\rho} \quad , \qquad \text{(A-19)}$$

where

$$T_{B\rho} = \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\frac{\partial U}{\partial \rho_i}\,\delta\rho_{Bim}\right]^2 \qquad \text{(A-20)}$$

and

$$T_{R\rho} = \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\frac{\partial U}{\partial \rho_i}\,\delta\rho_{Rim}\right]^2 \qquad \text{(A-21)}$$

After expansion and rearrangement of sums, Eq. (A-21) becomes

$$T_{R\rho} = \sum_{i=1}^{n}\left(\frac{\partial U}{\partial \rho_i}\right)^2\left[\frac{1}{M}\sum_{m=1}^{M}(\delta\rho_{Rim})^2\right] \quad , \qquad \text{(A-22)}$$

where the omitted cross terms again are zero. In Eq. (A-22) the expression in brackets is the variance $\sigma^2_{R\rho i}$ computed from the random deviations in the variable ρ_i appropriate to zone top i. Thus

$$T_{R\rho} = \sum_{i=1}^{n} \left(\frac{\partial U}{\partial \rho_i}\right)^2 \sigma_{R\rho i}^2 \quad . \tag{A-23}$$

When the square in Eq. (A-20) is expanded, the cross terms cannot be omitted because of the correlation of the $\delta\rho_{Bim}$ from zone to zone. Equation (A-20) becomes

$$T_{B\rho} = \sum_{i=1}^{n} \left(\frac{\partial U}{\partial \rho_i}\right)^2 \left[\frac{1}{M} \sum_{m=1}^{M} (\delta\rho_{Bim})^2 \right]$$
$$+ 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial U}{\partial \rho_i} \frac{\partial U}{\partial \zeta_j} \left[\frac{1}{M} \sum_{m=1}^{M} \delta\rho_{Bim} \, \delta\rho_{Bjm} \right] \quad . \tag{A-24}$$

If $\sigma_{B\rho i}^2$ is the bias contribution to the *variance* in ρ_i, then Eq. (A-24) may be written as

$$T_{B\rho} = \sum_{i=1}^{n} \left(\frac{\partial U}{\partial \rho_i}\right)^2 \sigma_{B\rho i}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial U}{\partial \rho_i} \frac{\partial U}{\partial \rho_j} r_{ij} \, \sigma_{B\rho i} \, \sigma_{B\rho j} \quad , \tag{A-25}$$

where r_{ij} is the correlation coefficient relating errors for zones i and j. We assume complete correlation for the bias errors, i.e., $r_{ij} = 1$. Under this condition the right side of Eq. (A-25) reduces to the square of the sum of terms in $\sigma_{B\rho i}$, yielding

$$T_{B\rho} = \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \sigma_{B\rho i} \right]^2 \quad . \tag{A-26}$$

From Eqs. (A-19), (A-23), and (A-26), we have

$$T_\rho = \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \sigma_{B\rho i} \right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \rho_i} \sigma_{R\rho i} \right]^2 \quad . \tag{A-27}$$

A similar expression can be obtained for T_ζ:

$$T_\zeta = \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \sigma_{B\zeta i} \right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \zeta_i} \sigma_{R\zeta i} \right]^2 \quad , \tag{A-28}$$

where $\sigma^2_{B\zeta i}$ and $\sigma^2_{R\zeta i}$ are, respectively, the bias and random contributions to the variance in ζ_i.

From Eqs. (A-10), (A-15), (A-16), (A-27), and (A-28), we express the variance in the East component of ballistic wind as

$$\begin{aligned} \sigma_U{}^2 = & \left[\frac{\partial U}{\partial \mu_0} \sigma_{R\mu 0} \right]^2 + \left[\frac{\partial U}{\partial v_0} \sigma_{Rv0} \right]^2 + \ldots \\ & + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \rho_i} \sigma_{B\rho i} \right]^2 + \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \zeta_i} \sigma_{B\zeta i} \right]^2 + \ldots \\ & + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \rho_i} \sigma_{R\rho i} \right]^2 + \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \zeta_i} \sigma_{R\zeta i} \right]^2 + \ldots \end{aligned} \tag{A-29}$$

The above result can be extended to any number of independent launch and zone variables. In particular, if we revert the variables μ_0, ν_0, ..., ρ_i, ζ_i, ... to the original generalized set λ_{10} λ_{20}, ..., λ_{L0}, ξ_{1i}, ξ_{2i}, ..., ξ_{Ki}, Eq. (A-29) becomes

$$\sigma_U{}^2 = \sum_{k=1}^{L} \left[\frac{\partial U}{\partial \lambda_{k0}} \sigma_{R\lambda k0} \right]^2 + \sum_{k=1}^{K} \left[\sum_{i=1}^{n} \frac{\partial U}{\partial \xi_{ki}} \sigma_{B\xi ki} \right]^2 + \sum_{k=1}^{K} \sum_{i=1}^{n} \left[\frac{\partial U}{\partial \xi_{ki}} \sigma_{R\xi ki} \right]^2 , \tag{A-30}$$

where we associate the standard deviation $\sigma_{R\lambda k0}$ with the random error in launch variable λ_{k0}. Similarly, we associate $\sigma_{B\xi ki}$ and $\sigma_{R\xi ki}$, respectively, with the bias and random errors in the zone variable ξ_{ki}. Equation (A-30) is the same as Eq. (4-3).

APPENDIX B. PARTIAL DERIVATIVES

In this appendix the computational forms of the partial derivatives required by the error models are given.

With reference to Figure 1, the following relationships can be written for the radiosonde at the top of zone i:

$$x_i = D_i \sin \alpha_i \tag{B-1}$$

and

$$y_i = D_i \cos \alpha_i \tag{B-2}$$

where each symbol retains its definition from Section 2.

Replacing the zone index i with the launch index 0 in Eqs. (B-1) and (B-2), we can evaluate the partial derivatives pertinent to the launch site. For example,

$$\frac{\partial x_0}{\partial D_0} = \sin \alpha_0 \quad . \tag{B-3}$$

All the required launch partials are given in Table XII.

The zonal partial derivatives $\frac{\partial x_i}{\partial t_i}$, $\frac{\partial x_i}{\partial \varepsilon_i}$, etc., $i = 1, 2, \ldots, n$, are evaluated assuming a spherical earth. To achieve this, any of several equivalent relationships among the geometric variables may be used as a starting point. From Reference 1 or 5, we write the distance D_i along the surface of the spherical earth as

$$D_i = R \arccos\left(\frac{R \cos \varepsilon_i}{R + z_i}\right) - R\varepsilon_i \quad , \tag{B-4}$$

where R is the mean radius of the earth and z_i is the altitude at the top of zone i. The elevation angle ε_i is expressed in radians.

From Eqs. (B-1), (B-2), and (B-4), we have

$$x_i = \left[R \arccos\left(\frac{R \cos \varepsilon_i}{R + z_i}\right) - R\varepsilon_i \right] \sin \alpha_i \quad , \tag{B-5}$$

and

$$y_i = \left[R \arccos\left(\frac{R \cos \varepsilon_i}{R + z_i}\right) - R\varepsilon_i \right] \cos \alpha_i \quad , \tag{B-6}$$

The required partial derivatives of x_i and y_i with respect to z_i, ε_i, and α_i are obtained from direct differentation of Eqs. (B-5) and (B-6). The results are given in Table XXI. In order to display the results compactly, it is convenient to define the following.

$$Q_i = \frac{R}{R + z_i} \tag{B-7}$$

and

$$\phi_i = \arccos\left(Q_i \cos \varepsilon_i\right) \quad . \tag{B-8}$$

The partial derivatives $\frac{\partial z_i}{\partial S_i}$ and $\frac{\partial z_i}{\partial \varepsilon_i}$ are required by the RADAR model. From Reference 6 the slant range S_i is given by

$$S_i = [(z_i + R)^2 + R^2 \cos^2 \varepsilon_i]^{\frac{1}{2}} - R \sin \varepsilon_i \tag{B-9}$$

We could solve this equation for z_i and proceed to evaluate the needed partials. However, it is somewhat easier to pursue the procedure outlined below.

We define the function $f(S_i, \varepsilon_i, z_i)$ to be

$$f(S_i, \varepsilon_i, z_i) = [(z_i + R)^2 - R^2 \cos^2 \varepsilon_i]^{\frac{1}{2}} - R \sin \varepsilon_i - S_i \tag{B-10}$$

Then from Eqs. (B-9) and (B-10),

$$f(S_i, \varepsilon_i, z_i) = 0 \quad . \tag{B-11}$$

The altitude z_i is taken to be dependent on S_i and ε_i.

Given the condition stated in Eq. (B-11), a theorem of partial differentiation allows us to write

$$\frac{\partial z_i}{\partial S_i} = - \frac{\partial f}{\partial S_i} \bigg/ \frac{\partial f}{\partial z_i} \tag{B-12}$$

and

$$\frac{\partial z_i}{\partial \varepsilon_i} = - \frac{\partial f}{\partial \varepsilon_i} \bigg/ \frac{\partial f}{\partial z_i} \quad , \tag{B-13}$$

when $\frac{\partial f}{\partial z_i}$ is not equal to zero. See Reference 7, Chapter 5, for example, for a discussion of this theorem.

The needed partial derivatives of f are evaluated using Eq. (B-10). Then $\frac{\partial z_i}{\partial S_i}$ and $\frac{\partial z_i}{\partial \varepsilon_i}$ are obtained from Eqs. (B-12) and (B-13), respectively. The results are displayed in Table XII.

Table XII. Partial Derivatives

$$\frac{\partial x_0}{\partial D_0} = \sin \alpha_0 \qquad \frac{\partial y_0}{\partial D_0} = \cos \alpha_0$$

$$\frac{\partial x_0}{\partial \alpha_0} = D_0 \cos \alpha_0 \qquad \frac{\partial y_0}{\partial \alpha_0} = - D_0 \sin \alpha_0$$

$$\frac{\partial x_i}{\partial z_i} = \frac{Q_i{}^2 \cos \varepsilon_i \sin \alpha_i}{\sin \phi_i} \qquad \frac{\partial y_i}{\partial z_i} = \frac{Q_i{}^2 \cos \varepsilon_i \cos \alpha_i}{\sin \phi_i}$$

$$\frac{\partial x_i}{\partial \varepsilon_i} = - R \sin \alpha_i \left[1 - \frac{Q_i \sin \varepsilon_i}{\sin \phi_i}\right] \qquad \frac{\partial y_i}{\partial \varepsilon_i} = - R \cos \alpha_i \left[1 - \frac{Q_i \sin \varepsilon_i}{\sin \phi_i}\right]$$

$$\frac{\partial x_i}{\partial \alpha_i} = R (\phi_i - \varepsilon_i) \cos \alpha_i \qquad \frac{\partial y_i}{\partial \alpha_i} = - R (\phi_i - \varepsilon_i) \sin \alpha_i$$

$$\frac{\partial z_i}{\partial S_i} = \sin \phi_i \qquad \frac{\partial z_i}{\partial \varepsilon_i} = R \sin (\phi_i - \varepsilon_i)$$

where R = mean radius of the earth

$$Q_i = \frac{R}{R + z_i}$$

$$\phi_i = \arccos (Q_i \cos \varepsilon_i)$$

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